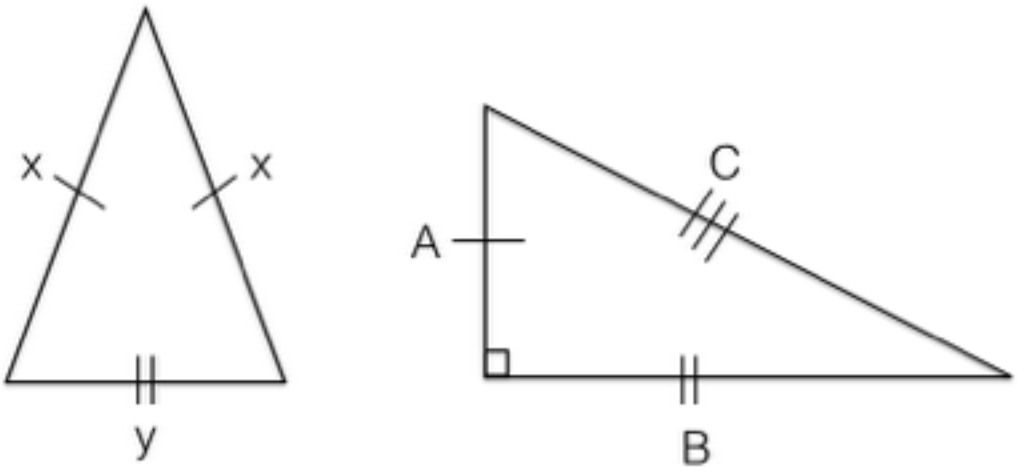
Oh No, Scarecrow! Math from the Wizard of Oz **by JW Gaberdiel**

At the end of The Wizard of Oz, the Scarecrow receives a diploma and then immediately says,

> *"The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side."*
>
> https://www.youtube.com/watch?v=DUCZXn9RZ9s

This is unfortunate. It sounds a lot like the Pythagorean Theorem:

> *"The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse."*



Isosceles:
Two legs congruent
Angles unknown
Scarecrow's Formula:
$\sqrt{x} + \sqrt{x} = \sqrt{y}$
or $\sqrt{x} + \sqrt{y} = \sqrt{x}$

Right:
One angle equal to 90
Sides unknown
Pythagoras' Formula:
$A^2 + B^2 = C^2$

However, Scarecrow's version is wildly and devastatingly different from Pythagoras' version.

First, the Pythagorean Theorem is about a right triangle (one angle is 90 degrees) not an isosceles triangle (two legs have the same length). In fact, once we complete the calculation, Scarecrow's description of an isosceles triangle is not even a triangle!

Second, let's take a look at what scarecrow has described. Scarecrow said the sum of the square roots of any two sides is equal to the square root of the remaining side. Thus, either:

Case 1: $\sqrt{x} + \sqrt{y} = \sqrt{x}$ or Case 2: $\sqrt{x} + \sqrt{x} = \sqrt{y}$

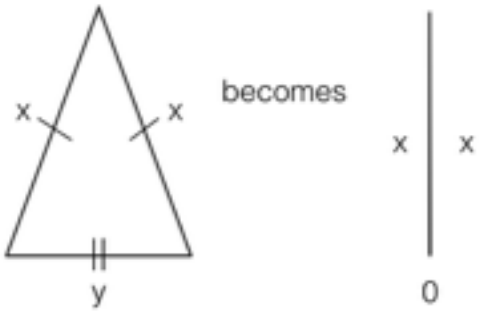
<u>In Case 1</u>, we can subtract √x from both sides:

$$\begin{array}{rr} \sqrt{x} + \sqrt{y} = \sqrt{x} \\ -\sqrt{x} \qquad -\sqrt{x} \end{array}$$

$$\sqrt{y} = 0$$

After squaring both sides we find

$$y = 0$$



This is an unfortunate result. It means that the bottom of the triangle has zero length, and the legs of the isosceles triangle collapse together like a pair of chopsticks and we are left with not a triangle at all, but a sad collapsed triangle which is really just a vertical straight line with length x.

<u>In Case 2</u>, we can add one √x plus one √x to equal two √x's:

$$\sqrt{x} + \sqrt{x} = \sqrt{y}$$

$$2\sqrt{x} = \sqrt{y}$$

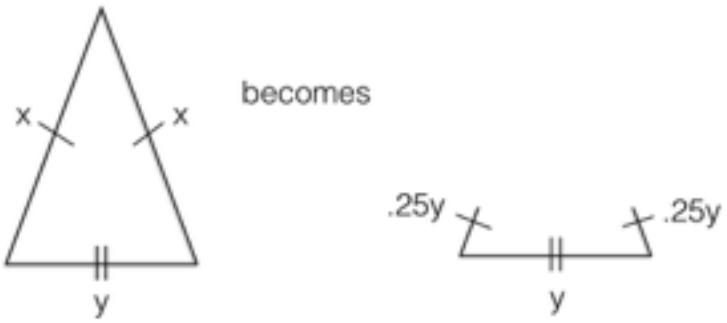
After squaring both sides we find

$$4x = y$$

Solve for x by dividing off the 4:

$$x = y/4$$

$$x = .25y$$



This result is even more tragic. If the congruent sides of the isosceles triangle are really each one-fourth of the length of the bottom, then their combined length (.25y + .25y = .5y) is not even long enough to reach across the bottom of the triangle. Thus, the congruent sides are left dangling in the air, disconnected.

<u>Conclusion</u>

I think we actually owe Scarecrow a mathematician's High Five for creating an incredibly subtle math joke. When you think about all of the <u>possible</u> triangles that can be built with three side lengths, you get 6 possibilities: scalene, isosceles, equilateral, acute, right, obtuse. However, when you think about all of the <u>impossible</u> triangles that cannot be built, you get 3 impossibilities: the collapsed triangle (Case 1), the disconnected triangle (Case 2), and the "degenerate triangle" (Case 3), in which all 3 sides have length equal to 0. In this third case of the degenerate triangle, all three sides shrink down to a single point in space. It's hardly a triangle, but it's worth pointing out that Scarecrow's Formula truthfully describes this third case as well. Just let all three values equal zero in either case, and the equation is True.

$$\begin{array}{ccc} \sqrt{x} + \sqrt{y} = \sqrt{x} & \text{or} & \sqrt{x} + \sqrt{x} = \sqrt{y} \\ \sqrt{0} + \sqrt{0} = \sqrt{0} & \text{or} & \sqrt{0} + \sqrt{0} = \sqrt{0} \\ 0 + 0 = 0 & \text{or} & 0 + 0 = 0 \\ 0 = 0 & & 0 = 0 \end{array}$$

Well done, Scarecrow. You created a formula that describes no real isosceles triangle ever.